MOAA 2025: Team Round

October 11th, 2025

Rules

- Your team has 40 minutes to complete 15 problems. Each answer is a nonnegative integer no greater than 1,000,000.
- If m and n are relatively prime, then the greatest common divisor of m and n is 1.
- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain and those of your teammates!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- Individuals may only receive help from members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, your team captain should write your answers down in the provided Team Round answer sheet. The proctors will collect your answer sheet immediately after the test ends.
- **Online:** Log into the Classtime session to access the test. Input all answers directly into the provided form. Select for the test to be handed in once you are ready.

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Team Round Problems

T1. **[5]** What is the remainder when $2025^{2026^{2027}}$ is divided by 10000?

T2. **[10]** In MOAA-land, each of the three letters M, O, and A cost a whole number of dollars. Given that

- 3 Ms are worth as much as 7 Os
- 8 Os are worth as much as 5 As

What is the minimum number of dollars that the word "MOAA" could cost?

T3. **[10]** If Brandon skips breakfast on a given day, then he eats breakfast the next day with probability p. Otherwise, if he eats breakfast on a given day, then he eats breakfast again the next day with probability $2p$. Given Brandon ate breakfast on Friday, and the probability that he ate breakfast two days later on Sunday was 72%, find $1000p$, rounded to the nearest integer.

T4. **[15]** Let $\triangle ABC$ be an equilateral triangle with $AB = 12$. A point P lies strictly inside the triangle such that the distance from P to BC is $\sqrt{3}$, and the areas of triangles PAB and PAC are in the ratio $2 : 3$. If S is equal to the area of triangle PCA, find S^2.

T5. **[15]** Anthony and Gentry are both driving from the same origin to the same destination. Anthony takes a 300-mile route, while Gentry takes a 400-mile route. Their driving speeds (in miles per hour) are chosen independently at random from the interval $[30, 60]$. Given the probability that Anthony arrives at the destination before Gentry can be written as $\frac{p}{q}$, where p and q are relatively prime integers, find $p + q$.

T6. **[20]** Jonjon has a broken clock which skips over any time showing the number 3. For example, after correctly showing the time 12:02, it will then show the time 12:04 even though it is actually 12:03. If Jonjon accurately sets his clock at 12:00, the time will actually be $a : bc$ when his clock first shows the time 6:00. What is $a + b + c$?

T7. **[20]** For any positive integer m, let $\tau(m)$ denote the number of divisors of m. How many positive integers n less than 2025 satisfy $\tau(\tau(n)) = 2$ and $\tau(n) \neq 2$?

T8. **[25]** Angela is standing on the bottom left square of a 3×3 grid. She wants to reach the top right square by moving only right or up one square at a time. Find the number of ways Bill can fill each square with the letter `a`, `b`, or `c`, such that every square contains exactly one letter and no matter what path Angela takes she will always encounter the letters `a`, `b`, `c` consecutively, in that order.

T9. **[30]** Let the *trimonic mean* of two positive integers a and b be $\frac{3}{\frac{1}{a}+\frac{1}{b}}$. Find the number of positive integers less than 100 that can be expressed as the trimonic mean of two distinct positive integers.

T10. **[35]** Let $k > 1$ be the unique real number such that the graph of $(|x| - k)^2 + (|y| - k)^2 = k$ in any 3 of the 4 quadrants share a common tangent line. The area of the region bound by the graph of $(|x|+k)^2+(|y|+k)^2 = k^4$ can be written in the form $\frac{a-b\sqrt{n}+c\pi}{d}$, where a, b, c, d, n are positive integers, n is square-free, and $\gcd(a, b, c, d) = 1$. Compute $a + bn + c + d$.

T11. **[35]** A number is constructed using only the digits 1 and 2. We consider a number to be "special" if the sum of its digits is 12. Two special numbers are considered equivalent if one can be obtained from the other by a cyclic shift of its digits. For example, 11221122 is equivalent to 12211221. How many non-equivalent special numbers are there?

T12. [**40**] Let $P(x)$ be a monic polynomial of degree 6 with integer coefficients satisfying

$$\begin{aligned} P(-1) &= 30, \\ P(3) &= 230, \\ P(-2) + P(4) &= 1770, \\ P(0) + P(2) &= 74. \end{aligned}$$

If $r_1, r_2, \ldots, r_6$ are the roots of $P(x)$, compute

$$(1 - r_1^2)(1 - r_2^2)(1 - r_3^2)(1 - r_4^2)(1 - r_5^2)(1 - r_6^2).$$

T13. [**45**] Let S be the set of integers n with $1 \leq n \leq 2000$ such that the greatest common divisor of $2^n + 1$ and $3^n + 1$ is divisible by 5 but not by 25, and such that $2^n + 1$ is divisible by 13. Compute the sum of all elements of S.

T14. [**45**] Consider the infinite sequence a_n consisting of only the positive integers 3 and 4, such that $a_1 = 3$, and a_n is equal to the number of 4's between the n^{th} and $(n+1)^{\text{st}}$ 3 in the sequence. For example, the next four terms are

$$a_2 = 4, \quad a_3 = 4, \quad a_4 = 4, \quad a_5 = 3.$$

Find the number of 3's in the first 2640 terms of the sequence.

T15. [**50**] Let ω_1, ω_2 be two circles that intersect at X and Y, and let A be a point on ω_1. Lines AX and AY intersect ω_2 at B and C, respectively, such that $AY = YC = 20$ and $AX = 25$. M is the midpoint of AB and N is the midpoint of AM. Denote by Ω the circle passing through M tangent to segments AB and BC. D is the tangency point of Ω to $\overline{BC}$, and $\overline{AD}$ intersects Ω again at E. Suppose that the perpendicular bisector of DE passes through the midpoint of BN. Then AE^2 can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.